## Tilt measurement using a low-g 3-axis accelerometer

## Introduction

This application note describes tilt sensing theory and the methods of determining tilt angle measurement of a low-g 3-axis accelerometer. In general, the procedures described here may also be applied to 3-axis analog or digital accelerometers, depending on their respective specifications.

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## 1 Calculating tilt angles

Low-g MEMS accelerometers are widely used for tilt sensing in consumer electronics and industrial applications, such as screen rotation and automobile security alert systems. Another popular application for low- $g$ accelerometers is tilt-compensated electronic compasses for map rotation and personal navigation devices. This application note describes how to obtain accurate tilt measurements with respect to local Earth horizontal plane, by compensating for a few non idealities that may cause angular tilt calculation error. For detailed information and device specifications, refer to the respective accelerometer datasheet available at http://www.st.com. In general, 3-axis analog or digital accelerometers may also be used, in accordance with their respective specifications.

### 1.1 Theory of operation

Figure 1 shows the single sensing axis of the accelerometer for tilt measurement.
Figure 1. Tilt measurement using a single axis of the accelerometer


The accelerometer measures the projection of the gravity vector on the sensing axis. The amplitude of the sensed acceleration changes according to the sine of the angle $\alpha$ between the sensing axis and the horizontal plane.

## Equation 1

$$
A=g \times \sin (\alpha)
$$

Using Equation 1, it is possible to estimate the tilt angle,

## Equation 2

$$
\alpha=\arcsin \left(\frac{A}{g}\right)
$$

where:

- $\mathrm{A}=$ acceleration measured
- $g=$ Earth's gravity vector

A single axis of the accelerometer with $360^{\circ}$ rotation is shown in Figure 2 and 3.
Figure 2. $360^{\circ}$ rotation of a single axis of the accelerometer


Figure 3. Plot of $360^{\circ}$ rotation of a single axis of the accelerometer


### 1.2 Tilt sensing

### 1.2.1 Single-axis tilt sensing

From Figure 2 and 3, it can be observed that the sensor is most responsive to changes in tilt angle when the sensing axis is perpendicular to the force of gravity. In this case, the sensitivity is approximately $17.45 \mathrm{mg} /{ }^{\circ}\left[=\sin \left(1^{\circ}\right)-\sin \left(0^{\circ}\right)\right]$. Due to the derivate function of the sine function, the sensor has lower sensitivity (less responsive to tilt angle changes) when the sensing axis is close to its +1 g or -1 g position. In this case, sensitivity is only $0.15 \mathrm{mg} /^{\circ}\left[=\sin \left(90^{\circ}\right)-\sin \left(89^{\circ}\right)\right]$. Table 1 shows the sensitivity at different tilt angles. In other words, the sine function has good linearity at [ $0^{\circ} 45^{\circ}$ ], [ $135^{\circ} 225^{\circ}$ ] and [ $315^{\circ} 360^{\circ}$ ] as shown in Figure 3.

Table 1. Tilt sensitivity of single axis accelerometer

| Tilt [ ${ }^{\circ}$ ] | Acceleration $[g]$ | $\Delta \boldsymbol{g} /^{\circ}\left[\mathrm{mg} /{ }^{\circ}\right]$ |
| :---: | :---: | :---: |
| 0 | 0.000 | 17.452 |
| 15 | 0.259 | 16.818 |
| 30 | 0.500 | 15.038 |
| 45 | 0.707 | 12.233 |
| 60 | 0.866 | 8.594 |
| 75 | 0.966 | 4.37 |
| 90 | 1.000 | 0.152 |

### 1.2.2 Dual-axis tilt sensing

When a dual-axis tilt sensing approach is used, the user should be aware of two different situations in which this approach could limit overall accuracy or even inhibit tilt calculation.

- Figure 4, Example A: Rotate the accelerometer counter-clockwise around the dotted arrow with $\beta$ angle. When $\beta$ is less than $45^{\circ}$, the $X$-axis has higher sensitivity, while the Y -axis has lower sensitivity. And when $\beta$ is greater than $45^{\circ}$, the X -axis has lower sensitivity while the Y -axis has higher sensitivity. Therefore, when the two-axis approach is used, it is always recommended to calculate the angle based on the orthogonal axis to a $\pm 1 \mathrm{~g}$ condition.
- Figure 4, Example B: At this position, both the X and Y axes have high sensitivity. However, without the help of a third axis (for example the Z -axis), it is impossible to distinguish a tilt angle of $30^{\circ}$ from one of $150^{\circ}$ because the X -axis has the same outputs at these two tilt angles.

Figure 4. Tilt sensitivity of a dual-axis accelerometer


### 1.2.3 Tri-axis tilt sensing

With a 3-axis accelerometer, the user can use the Z -axis to combine with the X and Y axes for tilt sensing in order to improve tilt sensitivity and accuracy (see Figure 5).

There are two ways to calculate 3 tilt angles in Figure 5. The first is to use basic trigonometric Equation 3, 4 and 5, where $\mathrm{A}_{\mathrm{x} 1}, \mathrm{~A}_{\mathrm{y} 1}$ and $\mathrm{A}_{\mathrm{z} 1}$ are the values obtained after applying accelerometer calibration on raw measurement data $\left(A_{x}, A_{y}, A_{z}\right)$ :

## Equation 3

$$
\alpha=\arcsin \left(\frac{A_{x 1}}{g}\right)
$$

## Equation 4

$$
\beta=\arcsin \left(\frac{A_{y 1}}{g}\right)
$$

## Equation 5

$$
\gamma=\arccos \left(\frac{A_{z 1}}{g}\right)
$$

Figure 5. Tilt angles from a tri-axis accelerometer


The second way is to use trigonometric Equation 6 and 7 to calculate pitch and roll tilt angle, which produces constant sensitivity over $360^{\circ}$ of rotation, as shown in Figure 6.

## Equation 6

$$
\text { Pitch }=\alpha=\arctan \left(\frac{\mathrm{A}_{\mathrm{x} 1}}{\sqrt{\left(\mathrm{~A}_{\mathrm{y} 1}\right)^{2}+\left(\mathrm{A}_{\mathrm{z} 1}\right)^{2}}}\right)
$$

## Equation 7

$$
\operatorname{Roll}=\beta=\arctan \left(\frac{\mathrm{A}_{\mathrm{y} 1}}{\sqrt{\left(\mathrm{~A}_{\mathrm{x} 1}\right)^{2}+\left(\mathrm{A}_{\mathrm{z} 1}\right)^{2}}}\right)
$$

Figure 6. Tilt sensitivity of a tri-axis accelerometer


## 2 Revision history

Table 2. Document revision history

| Date | Revision | Changes |
| :---: | :---: | :--- |
| 10-Jun-2014 | 1 | Initial release. |

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